

# Insights on Oscillations and Impulses in Variable Impedance Control

Giorgio Simonini<sup>1</sup>, Rachele Nebbia Colomba<sup>1</sup>, Chiara Sammarco<sup>1</sup>,  
Mathew J. Pollayil<sup>1</sup>, Franco Angelini<sup>1</sup>, Antonio Bicchi<sup>1,2</sup>

**Abstract**—Operational-space impedance control allows to assign a mass-damper-spring (MDS) behavior to the end effector of a robotic manipulator. This aids resilience when interactions with the environment or humans are expected. While changing the virtual mass is not always practicable since it requires using precise force/torque sensors, varying the stiffness and damping can be useful to intuitively modify the response of the robot to external perturbations. In this paper, considerations are made on MDS systems regarding oscillations and impulse responses of the system when its impedance is varied. These observations are then validated in simulation using a two degrees of freedom (DoFs) robotic arm.

## I. INTRODUCTION

Robots are increasingly present in everyday life [1]; the so-called cobots allow tasks to be performed in collaboration with humans while reducing the risk of harming them. Tasks involving robot-environment or human-robot interactions, e.g. grasping or collaboration logistics, are the current focus of the so-called Industry 4.0 and 5.0 and the general trend is to enhance safety by using robots that are not rigid. There are mainly two ways, passive and active [2]. The former integrates elastic elements in the robot body and concerns soft robotics [3]. The latter uses the control strategy to achieve compliance in the robot. The one considered in this paper is the impedance control [4] which makes it possible to generate torques at the joints that simulate a mass-damper-spring (MDS) system on the end effector side.

As humans, we are able to vary the body stiffness during our movements, and this suggests that modifying online the impedance of the robotic system could lead to significant benefits [5], [6]. Moreover, changing the stiffness and damping allows intuitive management of the behavior at the end effector; as an example, the authors of [7] use variable impedance control to adapt the contact force with the environment. In this paper, we bring reasoning regarding a single-degree-of-freedom MDS system. We analyze the behavior by looking at what could be achieved by varying the impedance coefficients. Finally, we validated the results in simulation by recreating through impedance control the dynamics of an MDS system at the end effector.

This work was supported in part by the European Union’s Horizon 2020 Research and Innovation Program under Grant Agreements No. 101017274 (Darko) and No. 101016970 (Natural Intelligence) and in part by the Ministry of University and Research (MUR) as part of the PON 2014-2021 “Research and Innovation” resources – Green Action - DM MUR 1062/2021.

<sup>1</sup>Dipartimento di Ingegneria dell’Informazione e Centro di Ricerca “Enrico Piaggio”, Università di Pisa, Largo Lucio Lazzarino 1, 56126 Pisa, Italy

<sup>2</sup>Soft Robotics for Human Cooperation and Rehabilitation, Fondazione Istituto Italiano di Tecnologia, via Morego, 30, 16163 Genova, Italy  
giorgio.simonini.work@gmail.com

## II. IMPEDANCE CONTROL BACKGROUND

Consider the well-known dynamic equation for rigid manipulators

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau, \quad (1)$$

with  $B \in \mathbb{R}^{n \times n}$  the mass matrix,  $C \in \mathbb{R}^{n \times n}$  the Coriolis and centrifugal matrix,  $G \in \mathbb{R}^n$  the gravity terms,  $\tau \in \mathbb{R}^n$  the generalized torque, and  $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$  the link positions, velocities, and accelerations respectively.

Consider also the mono-dimensional second order dynamics

$$m\ddot{x} + d\dot{x} + kx = F_{\text{ext}}, \quad (2)$$

with  $m, d, k \in \mathbb{R}$  the inertia, damping, and stiffness factors,  $F_{\text{ext}} \in \mathbb{R}$  the external force and  $x, \dot{x}, \ddot{x} \in \mathbb{R}$  the position, velocity, and acceleration respectively. As described in [8] the impedance control can bring to the end effector the desired acceleration of an MDS system in the task space. Furthermore, a proper selection of the gains can make the virtual system uncoupled and it permits to reason on a single dimensional system. Therefore, the following considerations are made on the system (2), keeping in mind that the control law uses a fixed position reference.

## III. MDS CONSIDERATIONS

This section presents some interesting results related to behaviors that can be achieved by adequately varying the impedance. More in detail, we show how attenuation or accretion of oscillations can be obtained through appropriate variation of stiffness. Damping, instead, can shape the transient response of impulsive events.

a) *Antibouncing*: Assume that the virtual system cannot be made critically damped nor overdamped for some reason. The system will initially have virtual potential energy  $E = \frac{1}{2}kx^2$ . Using the stiffness update law

$$k = k_0 + \gamma_1 x \dot{x}, \quad (3)$$

with  $\gamma_1 \in \mathbb{R}$  a positive parameter, we obtain that the virtual energy stored in the spring decreases over oscillation cycles, and thus the system tends toward equilibrium faster than the one with constant stiffness. Note that this law reduces oscillations when the system is underdamped. Sure enough, if the system is critically damped ( $d = 2\sqrt{km}$ ) or overdamped, oscillations do not occur and the method in (3) is not necessary.

b) *Instability*: Here, the question arises as to whether it is possible to give to the system (2) an unstable behavior by varying stiffness in a similar fashion. Using the update law

$$k = k_0 - \gamma_2 \text{sign}(x\dot{x}), \quad (4)$$

the stiffness is increased when the energy stored in the spring is maximum, while it is decreased at the minimum. This ensures instability if the energy gained is greater than that consumed by the damping. An intuitive explanation is that the virtual mass is called back with greater force than it is braked, increasing its energy. Also this situation is obviously possible only in underdamped systems.

c) *Impulse response*: Suppose a shock occurs at the end effector. This is an impulse that can be represented as a change in initial conditions with non-zero velocity. In this situation, one would like the robot to return as quickly as possible and without oscillations to the equilibrium configuration. With this in mind, at least a critically damped system is needed. Indeed, critical damping is usually used to avoid overshoots or oscillations. However, in the case of impulses, the use of a fixed impedance slows down the dynamics and it can not be the fastest choice. It is useful to use a two-phase damping

$$d = \begin{cases} \gamma_3 \sqrt{km} & \text{if } |\dot{x}| > 0, \\ 2\sqrt{km} & \text{if } |\dot{x}| \leq 0, \end{cases} \quad (5)$$

with  $\gamma_3 \in \mathbb{R} > 2$  to obtain an overamped behavior. If the damping profile is taken according to (5), we have that the system in the first phase dissipates more virtual energy by being overdamped. Once zero velocity is reached, the system is made critically damped so that it returns to the equilibrium condition as quickly as possible without overshoot.

#### IV. VALIDATION

The situations presented in the previous section were simulated on a two DoFs robotic arm. Regarding *antibouncing* (Fig. 1(a), 1(b)), the system started with a displacement with respect to the equilibrium position, and it can be seen how a variable stiffness yields a better response in reaching the desired point. Fig. 1(b) represents the variable stiffness as (3).

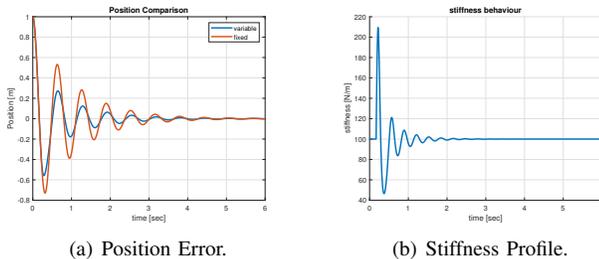


Fig. 1: Antibouncing: Comparison between fixed and variable stiffness on initial displacement. Using the stiffness profile in (b) the oscillations are damped out (a).

The impedance law (4), on the other hand, allows instability to be reached quickly (Fig. 2(a)). It can be seen also that the energy of the system increases over time (Fig. 2(b)).

Finally, Fig. 3 shows the impulse response on the robot's end effector. Notice that the fastest response is achieved using variable damping.

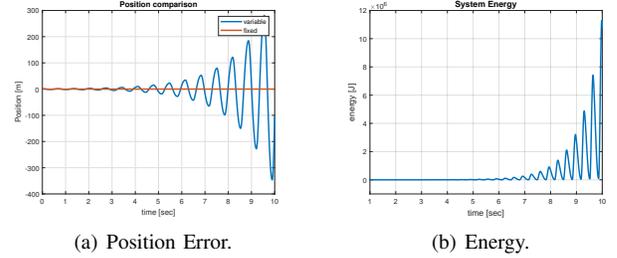


Fig. 2: Instability: Comparison between fixed and variable stiffness on initial displacement. Using the stiffness profile in (b) the system achieves the desired unstable behavior (a).

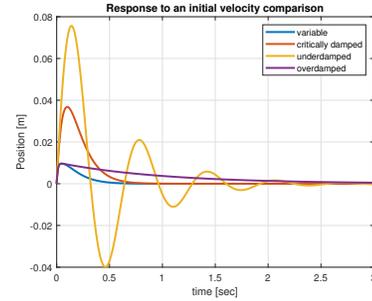


Fig. 3: Impulse response: Comparison of different damping profiles. The fastest one to damp out the disturbance without oscillations is the variable one obtained through (5). Note that it is even faster than the critically damped case.

#### V. CONCLUSIONS

Impedance control seeks to realize a specific impedance of the robot. Varying the stiffness and damping profiles allows to change the behavior of the system intuitively. We explored this possibility in relation with three different scenarios: oscillation attenuation (*Antibouncing*), oscillation accretion (*Instability*), and fast impulse convergence (*Impulse response*). Results are general and these considerations could be used also in systems not driven by impedance control, for example in soft robotics and variable stiffness actuation or car suspension systems as well.

#### REFERENCES

- [1] K. S. D. Zoltan Dobra, "Technology jump in the industry: human–robot cooperation in production," *Emerald Publishing Limited*, 2020.
- [2] L. R. Wang W. and G. E.Y., "Passive compliance versus active compliance in robot-based automated assembly systems," *Industrial Robot*, vol. 25, pp. 48–57, 1998.
- [3] C. Della Santina, M. G. Catalano, and A. Bicchi, "Soft robots," *Encyclopedia of Robotics*, vol. 489, 2020.
- [4] N. Hogan, "Impedance control: An approach to manipulation," in *1984 American Control Conference*, 1984, pp. 304–313.
- [5] R. Ikeura and H. Inooka, "Variable impedance control of a robot for cooperation with a human," in *Proceedings of 1995 IEEE International Conference on Robotics and Automation*, vol. 3, 1995, 3097–3102 vol.3.
- [6] F. Ficuciello, L. Villani, and B. Siciliano, "Variable impedance control of redundant manipulators for intuitive human–robot physical interaction," *IEEE Transactions on Robotics*, vol. 31, no. 4, pp. 850–863, 2015.
- [7] K. Lee and M. Buss, "Force tracking impedance control with variable target stiffness," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 6751–6756, 2008, 17th IFAC World Congress, ISSN: 1474-6670.
- [8] C. Ott, "Cartesian impedance control: The rigid body case," in *Cartesian Impedance Control of Redundant and Flexible-Joint Robots*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2008, pp. 29–44, ISBN: 978-3-540-69255-3.